Universidade Federal de Campina Grande - UFCG
Centro de Ciências e Tecnologias - CCT
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Disciplina: Cálculo Diferencial e Integral I

Lista 2 - Derivadas

1 - Use a definição de derivada pra calcular a derivada de cada uma das funções no ponto indicado:

a) $f(x) = 5x - 3$, em $a = -3$.

b) $f(x) = x^2 + x$, em $a = 1$

c) $f(x) = x^2 + 3x - 5$, em $a = -2$.

d) $f(x) = 4x^5 - x^4 - 3x^2 - 2$, em $a = 1$.

e) $f(x) = (x + x^3)(x^5 - 3x^2 + x)$, em $a = 0$.

f) $f(x) = \dfrac{1-x}{2+x}$, em $a = 0$.

g) $f(x) = \sqrt{x}$, em $a = 4$.

h) $f(x) = \dfrac{1}{x}$, em $a = 1$.

i) $f(x) = \dfrac{1}{x^2}$, em $a = 2$.

j) $f(x) = \dfrac{1}{x^2 - 3x}$, em $a = 2$.

2 - Determine a equação da reta tangente ao gráfico de cada função a seguir no ponto a indicado.

a) $f(x) = 3x - x^2$, em $a = 2$.

b) $f(x) = x^3 - 4x^2 + 5$, em $a = 0$.

c) $f(x) = 2x^3 - x^2 - 4x$, em $a = -1$.

d) $f(x) = x^5 - 4x^3 - 2$, em $a = 1$.

3 - Determine a equação da reta normal ao gráfico de cada uma das funções do exercício anterior, no mesmo ponto a indicado.

4 - Determine a(s) reta(s) tangente(s) ao gráfico da função $f(x)$ paralela(s) à reta r dada:

a) $f(x) = 4x^2 - 5$ $\qquad r : y = 4x + 3$.

b) $f(x) = x^4 - 6x^2 + x \qquad r: 7x + y - 4 = 0.$

5 - Determine a(s) reta(s) tangente(s) ao gráfico da função $f(x)$ perpendiculares(s) à reta r dada:

a) $f(x) = x^2 + 2x - 1 \qquad r: x + 2y - 5 = 0.$

b) $f(x) = x^3 + 6x - 3 \qquad r: x + 18y + 3 = 0.$

6 - Seja $f(x) = x^2$. Determine a equação da reta que é tangente ao gráfico de f e paralela a reta $y = \frac{1}{2}x + 3$.

7 - Sabe-se que r é uma reta tangente ao gráfico de $f(x) = x^3 + 3x$ e paralela à reta $y = 6x - 1$. Determine r.

8 - Determine a equação da reta que é perpendicular à reta $2y + x - 3 = 0$ e tangente as gráfico de $f(x) = x^2 - 3x$.

9 - Para cada equação determine $\frac{dy}{dx}$.

a) $(x-1)^2 + y^2 = 3$

b) $x^3 - (y+3)^2 = xy$

c) $(5-x)^2 + xy = x$

d) $(x-2)^3 + xy = y^3$

e) $y^2 = \frac{x-1}{x+1}$

f) $x = \text{tg} y$

g) $x + \text{tg}(xy) = 0$

h) $e^{2x} = \text{sen}(x + 3y)$

10 - Determine a equação da reta tangente a cada curva no ponto p indicado:

a) $xy + y^2 = 2$, em $p = (1, 1)$

b) $3x^2 - (1-y)^2 = x + 1$, em $p = (2, -2)$

c) $(x-3)^2 + (y-1)^2 = 17$, em $p = (-1, 2)$

d) $4x - 3xy^2 + x^2y = 0$, em $p = (1, -1)$

e) $x^3 - xy + y^3 = 7$, em $p = (2, 1)$

11 - Suponha que $y = f(x)$ seja uma função derivável e dada implicitamente pela equação

$$xy^2 + y + x = 1.$$

Mostre que $f'(x) = \frac{-1 - [f(x)]^2}{2xf(x) + 1}$ em todo x no domínio de f com $2xf(x) + 1 \neq 0$.

12 - A função $y = f(x)$, com $y > 0$, é dada implicitamente por $x^2 + 4y^2 = 2$. Determine a equação da reta tangente ao gráfico de f, no ponto de abscissa 1.

13 - Expresse $\dfrac{dy}{dx}$ em termos de x e y, onde $y = f(x)$ é uma função derivável dada implicitamente pela equação:

a) $xe^y + xy = 3$

b) $5y + \cos y = xy$

c) $y + \ln(x^2 + y^2) = 4$

d) $2y + \text{sen}y = x$

e) $xy + y^3 = x$

f) $x^2y^3 + xy = 2$

14 - Derive:

a) $y = \text{sen}(4x)$

b) $y = \cos(5x)$

c) $y = e^{3x}$

d) $y = \text{sen}x^3$

e) $y = \ln(2x + 1)$

f) $y = e^{\text{sen}x}$

g) $y = \cos(e^x)$

h) $y = (\text{sen}x + \cos x)^3$

i) $y = \sqrt{3x + 1}$

j) $y = \ln(x^2 + 3x + 9)$

k) $y = e^{-5x}$

l) $y = \text{sen}(\cos x)$

m) $y = e^{\text{tg}x}$

n) $y = \cos(x^2 + 3)$

o) $y = \sec 3x$

p) $y = xe^{3x}$

q) $y = e^x\cos(2x)$

r) $y = e^{-x^2} + \ln(2x + 1)$

s) $y = \dfrac{\cos 5x}{\text{sen}2x}$

t) $y = x\ln(2x + 1)$

u) $y = (\ln(x^2 + 1))^3$

v) $y = \ln(\cos(2x))$

15 - Calcule a derivada das funções trigonométricas:

a) $y = \text{tg}x$

b) $y = \sec x$

c) $y = \text{cotg}x$

d) $y = \text{cosec}x$

16 - Derive:

$a)\ y = \mathrm{tg}(3x)$ $b)\ y = \sec(4x)$ $c)\ y = \mathrm{cotg}(x^2)$

$d)\ y = \sec(\mathrm{tg}x)$ $e)\ y = \sec(x^3)$ $f)\ y = e^{\mathrm{tg}(x^2)}$

$g)\ y = \mathrm{cosec}(2x)$ $h)\ y = x^3\mathrm{tg}(4x)$ $i)\ y = \ln(\sec(3x) + \mathrm{tg}(3x))$

$j)\ y = e^{-x}\sec(x^2)$ $k)\ y = (x^2 + \mathrm{cotg}(x^2))^3$ $l)\ y = x^2\mathrm{tg}(2x)$

17 - Determine a derivada:

$a)\ y = \mathrm{arcsen}3x$ $b)\ y = \mathrm{arctg}(2x+3)$

$c)\ y = \mathrm{arcsen}(e^x)$ $d)\ y = e^{3x}\mathrm{arcsen}2x$

$e)\ y = \dfrac{\mathrm{sen}3x}{\mathrm{arctg}(4x)}$ $f)\ y = x^2e^{\mathrm{arctg}2x}$

$g)\ y = \dfrac{x\mathrm{arctg}x}{\cos 2x}$ $h)\ y = e^{-3x} + \ln(\mathrm{arctg}x)$

$i)\ f(x) = \mathrm{arcsen}(e^x)$ $j)\ f(x) = e^x\arccos(x^2)$

$k)\ f(x) = x^2\mathrm{arctg}(4x)$ $l)\ f(x) = \arccos(e^x + 1)$

$m)\ f(x) = e^x + \mathrm{arcsen}(x^2)$ $n)\ f(x) = \mathrm{arctg}^2(\cos x)$

18 - Use a derivação logarítmica para determinar $\dfrac{dy}{dx}$:

$a)\ y = (x+1)^x$ $b)\ y = (\mathrm{sen}x)^x$

$c)\ y = x^{\mathrm{sen}x}$ $d)\ y = x^{\ln(x)}$

$e)\ y = (x+2)^x$ $f)\ y = (1+e^x)^{x^2}$

$g)\ y = (4 + \mathrm{sen}(3x))^x$ $h)\ y = (x+3)^{x^2}$

$i)\ y = (3x+\pi)^{x^2}$ $j)\ y = (x^2+1)^\pi$

$k)\ f(x) = (\cos x)^{e^x}$ $l)\ f(x) = (x+1)^{\cos x}$

$m)\ f(x) = (\mathrm{sen}x)^{\cos x}$ $n)\ f(x) = (2x+1)^{\ln x}$

19 - Use derivação logarítmica para calcular:

$a)\ y = \sqrt{x(x+1)}$

$b)\ y = \sqrt{(x^2+1)(x-1)^2}$

$c)\ y = \sqrt{\dfrac{1}{x(x+1)}}$

$d)\ y = \sqrt[3]{\dfrac{x(x-2)}{x^2+1}}$

$e)\ y = \sqrt{\dfrac{(x+1)^{10}}{(2x+1)^5}}$

$f)\ y = \sqrt[3]{\dfrac{x(x+1)(x-2)}{(x^2+1)(2x+3)}}$

20 - Derive.

$a)\ y = \text{sen}(\text{tg}(2x))$

$b)\ y = 2^{\text{sen}(\pi x)}$

$c)\ f(s) = \sqrt{\dfrac{s^2+1}{s^2+4}}$

$d)\ y = \text{sen}^2(e^{\text{sen}^2(x)})$

$e)\ y = 2^{3^{x^2}}$

$f)\ y = x^2 e^{-1/x}$

$g)\ f(x) = \text{sen}(\text{sen}x)$

$h)\ f(v) = \left(\dfrac{v}{v^3+1}\right)^6$

$i)\ f(x) = \dfrac{\cos x}{x^2+1}$

$j)\ f(x) = \dfrac{x+1}{x\text{sen}x}$

$k)\ f(x) = \dfrac{1+e^{3x}}{1-e^{5x}}$

$l)\ f(x) = \dfrac{x+1}{x\ln x}$

$m)\ f(x) = e^{-x}\text{sen}(2x)$

$n)\ f(x) = x^3 e^{-3x}$

$o)\ f(x) = \cos^3 x^3$

$p)\ f(x) = (\ln(x^2+1))^3$

$q)\ f(x) = x^3\text{tg}(4x)$

$r)\ f(x) = \sqrt{x\sec(x^2+2)}$

$s)\ f(x) = \dfrac{xe^{2x}}{\ln(3x+1)}$

$t)\ f(x) = \dfrac{xe^{-2x}}{\sec(3x)}$

$u)\ f(x) = x5^{x^2}$

$v)\ f(x) = \dfrac{x^2+1}{\sqrt{x+1}}$

21 - Seja $y = \dfrac{1}{x^2}$. Verifique que $x\dfrac{dy}{dx} + 2y = 0$.

22 - Seja $y = -\dfrac{2}{x^2+k}$, k constante. Verifique que $\dfrac{dy}{dx} - xy^2 = 0$.

23 - Seja $y = \cos x$. Verifique que $\dfrac{d^2y}{dx^2} + y = 0$.

24 - Seja $y = e^x \cos x$. Verifique que $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 2y = 0$.

GABARITO

01. $a)$ 5 $\quad b)$3 $\quad c)$ -1 $\quad d)$ 10 $\quad e)$ 0 $\quad f)$ $-3/4$ $\quad g)$ $1/4$ $\quad h)$ -1 $\quad i)$ $-1/4$ $\quad j)$ $-\frac{1}{4}$

02. (a) $y = -x + 4$ $\quad (b)$ $y = 5$ $\quad (c)$ $y = 4x + 5$ $\quad (d)$ $y = -7x + 2$

03. (a) $y = x$ $\quad (b)$ $x = 0$ $\quad (c)$ $y = -\dfrac{x}{4} + \dfrac{3}{4}$ $\quad (d)$ $y = \dfrac{x}{7} - \dfrac{36}{7}$

04. (a) $y = 4x - 6$ $\quad (b)$ $y = -7x - 24$ e $y = -7x + 3$.

05. (a) $y = 2x - 1$ $\quad (b)$ $y = 18x - 19$ e $y = 18x + 13$.

06. $y = \dfrac{x}{2} - \dfrac{1}{16}$

07. $y = 6x - 2$ ou $y = 6x + 2$

08. $y = 2x - \dfrac{25}{4}$

09. (a) $\dfrac{-x+1}{y}$ $\quad (b)$ $\dfrac{3x^2 - y}{x + 2y + 6}$ $\quad (c)$ $\dfrac{11 - 2x - y}{x}$ $\quad (d)$ $\dfrac{3(x-2)^2 + y}{3y^2 - x}$ $\quad (e)$ $\dfrac{1}{y(x+1)^2}$
(f) $\cos^2 y$ $\quad (g)$ $\dfrac{-\cos^2(xy) - y}{x}$ $\quad (h)$ $\dfrac{2e^{2x} - \cos(x + 3y)}{3\cos(x + 3y)}$

10. $a)$ $y = -\dfrac{x}{3} + \dfrac{4}{3}$ $\quad b)$ $y = -\dfrac{11x}{6} + \dfrac{5}{3}$ $\quad (c)$ $y = 4x + 6$ $\quad (d)$ $y = \dfrac{x}{7} - \dfrac{8}{7}$
(e) $y = -11x + 23$

12. $y = -\dfrac{x}{2} + 1$

13. $a)$ $\dfrac{dy}{dx} = \dfrac{-y - e^y}{xe^y + x}$ $\quad b)$ $\dfrac{dy}{dx} = \dfrac{y}{5 - \text{sen}y - x}$ $\quad c)$ $\dfrac{dy}{dx} = \dfrac{-2x}{x^2 + y^2 + 2y}$

$d)$ $\dfrac{dy}{dx} = \dfrac{1}{2 + \cos y}$ $\quad e)$ $\dfrac{dy}{dx} = \dfrac{1 - y}{x + 3y^2}$ $\quad f)$ $\dfrac{dy}{dx} = \dfrac{-y - 2xy^3}{x + 3x^2y^2}$

14. (a) $4\cos(4x)$ $\quad (b)$ $-5\text{sen}(5x)$ $\quad (c)$ $3e^{3x}$ $\quad (d)$ $3x^2\cos(x^3)$ $\quad (e)$ $\dfrac{2}{2x+1}$
(f) $e^{\text{sen}x}\cos x$ $\quad (g)$ $-e^x\text{sen}(e^x)$ $\quad (h)$ $3(\text{sen}x + \cos x)^2(\cos x - \text{sen}x)$ $\quad (i)$ $\dfrac{3}{2\sqrt{3x+1}}$

(j) $\dfrac{2x + 3}{x^2 + 3x + 9}$ $\quad (k)$ $-5e^{-5x}$ $\quad (l)$ $-\cos(\cos x)\text{sen}x$ $\quad (m)$ $e^{\text{tg}x}\sec^2 x$ $\quad (n)$ $-2x\text{sen}(x^2 + 3)$ $\quad (o)$ $3\text{tg}(3x)\sec(3x)$ $\quad (p)$ $e^{3x}(1+3x)$ $\quad (q)$ $e^x(\cos(2x) - 2\text{sen}(2x))$ $\quad (r)$ $-2xe^{-x^2} + \dfrac{2}{2x+1}$ $\quad (s)$ $\dfrac{-5\text{sen}(5x)\text{sen}(2x) - \cos(5x)\cos(2x)}{\text{sen}^2(2x)}$ $\quad (t)$ $\ln(2x+1) + \dfrac{2x}{2x+1}$ $\quad (u)$ $\dfrac{6x(\ln(x^2+1))^2}{x^2+1}$

(v) $-2\text{tg}(2x)$

15. (a) $y = \sec^2(x)$ (b) $y = \text{tg}(x)\sec(x)$ (c) $y = -\text{cosec}^2(x)$ (d) $y = -\text{cotg}(x)\text{cosec}(x)$

16. (a) $3\sec^2(3x)$ (b) $4\text{tg}(4x)\sec(4x)$ (c) $-2x\text{cosec}^2(x^2)$ (d) $\text{tg}(\text{tg}x)\sec(\text{tg}x)\sec^2(x)$ (e) $3x^2\text{tg}(x^3)\sec(x^3)$ (f) $2xe^{\text{tg}(x^2)}\sec^2(x^2)$ (g) $-2\text{cosec}(2x)\text{cotg}(2x)$ (h) $3x^2\text{tg}(4x)+4x^3\sec^2(4x)$ (i) $3\sec(3x)$ (j) $e^{-x}\sec(x^2)(2x\text{tg}(x^2)-1)$ (k) $6x(x^2+\text{cotg}(x^2))^2(1-\text{cosec}^2(x^2))$ (l) $2x(\text{tg}(2x)+x\sec^2(2x))$

17. (a) $y' = \dfrac{3}{\sqrt{1-9x^2}}$ (b) $y' = \dfrac{2}{1+(2x+3)^2}$ (c) $y' = \dfrac{e^x}{\sqrt{1-e^{2x}}}$

(d) $y' = e^{3x}\left(3\text{arcsen}2x + \dfrac{2}{\sqrt{1-4x^2}}\right)$ (e) $y' = \dfrac{3\cos 3x\ \text{arctg}4x\ (1+16x^2) - 4\text{sen}\ 3x}{(1+16x^2)(\text{arctg}\ 4x)^2}$

(f) $y' = 2xe^{\text{arctg}2x}\left(1+\dfrac{x}{1+4x^2}\right)$ $y' = \dfrac{\left(\text{arctg}x + \dfrac{x}{1+x^2}\right)\cos 2x - 2x\text{arctg}x\ \cos 2x}{(\cos 2x)^2}$

(h) $y' = -3e^{-3x} + \dfrac{1}{(1+x^2)\text{arctg}x}$ $i)$ $\dfrac{e^x}{\sqrt{1-e^{2x}}}$ $j)$ $e^x\left(\arccos(x^2) - \dfrac{2x}{\sqrt{1-x^4}}\right)$

$k)$ $2x\text{arctg}(4x) + \dfrac{4x^2}{1+16x^2}$ $l)$ $\dfrac{-e^x}{\sqrt{1-(e^x+1)^2}}$ $m)$ $e^x + \dfrac{2x}{\sqrt{1-x^4}}$ $n)$ $\dfrac{2\text{sen}x\cdot\text{arctg}(\cos x)}{1+\cos^2 x}$

18. (a) $(x+1)^x\left(\ln(x+1)+\dfrac{x}{x+1}\right)$ (b) $(\text{sen}x)^x(\ln(\text{sen}x)+x\text{cotg}x)$

(c) $x^{\text{sen}x}\left(\cos x\ln x + \dfrac{\text{sen}x}{x}\right)$ (d) $x^{\ln x}\left(\dfrac{\ln(x^2)}{x}\right)$ (e) $(x+2)^x\left(\ln(x+2)+\dfrac{x}{x+2}\right)$

(f) $(1+e^x)^{x^2}\left(2x\ln(1+e^x)+\dfrac{x^2e^x}{1+e^x}\right)$ (g) $(4+\text{sen}(3x))^x\left(\ln(4+\text{sen}(3x))+\dfrac{3x\cos(3x)}{4+\text{sen}(3x)}\right)$

(h) $(x+3)^{x^2}\left(2x\ln(x+3)+\dfrac{x^2}{x+3}\right)$ (i) $(3x+\pi)^{x^2}\left(2x\ln(3x+\pi)+\dfrac{3x^2}{3x+\pi}\right)$

(j) $\dfrac{2\pi x(x^2+1)^\pi}{x^2+1}$ (k) $(\cos x)^{e^x}(e^x\ln(\cos x)-e^x\text{tg}x)$ (l) $(x+1)^{\cos x}\left(\dfrac{\cos x}{x+1}-\text{sen}x\ln(x+1)\right)$

(m) $(\text{sen}x)^{\cos x}(\cos x\text{cotg}x - \text{sen}x\ln(\text{sen}x))$ (n) $(2x+1)^{\ln x}\left(\dfrac{\ln(2x+1)}{x}+\dfrac{2\ln x}{2x+1}\right)$

19. $a)$ $\dfrac{dy}{dx} = \sqrt{x(x+1)}\left(\dfrac{1}{2x}+\dfrac{1}{2(x+1)}\right)$ $b)$ $\dfrac{dy}{dx} = \sqrt{(x^2+1)(x-1)^2}\left(\dfrac{x}{x^2+1}+\dfrac{1}{x-1}\right)$

$c)$ $\dfrac{dy}{dx} = -\dfrac{1}{2}\sqrt{\dfrac{1}{x(x+1)}}\left(\dfrac{1}{x}+\dfrac{1}{x+1}\right)$ $d)$ $\dfrac{dy}{dx} = \dfrac{1}{3}\sqrt[3]{\dfrac{x(x-2)}{x^2+1}}\left(\dfrac{1}{x}+\dfrac{1}{x-2}-\dfrac{2x}{x^2+1}\right)$

$d)\ \dfrac{dy}{dx} = \sqrt{\dfrac{(x+1)^{10}}{(2x+1)^5}} \left(\dfrac{5}{x+1} - \dfrac{5}{2x+1} \right)$

$e)\ \dfrac{dy}{dx} = \dfrac{1}{3} \sqrt[3]{\dfrac{x(x+1)(x-2)}{(x^2+1)(2x+3)}} \left(\dfrac{1}{x} + \dfrac{1}{x+1} + \dfrac{1}{x-2} - \dfrac{2x}{x^2+1} - \dfrac{2}{2x+3} \right)$

20. $a)\ 2\cos(\mathrm{tg}2x)\sec^2(2x)$ $\quad b)\ \pi \ln 2 \cos(\pi x) 2^{\mathrm{sen}(\pi x)}$ $\quad c)\ \dfrac{3s}{(s^2+4)^2} \left(\dfrac{s^2+1}{s^2+4} \right)^{-1/2}$

$d)\ 4\mathrm{sen}(e^{\mathrm{sen}^2(x)}) \cos(e^{\mathrm{sen}^2(x)}) e^{\mathrm{sen}^2(x)} \mathrm{sen} x \cos x$ $\quad e) 2x \ln 2 \ln 3 (2^{3^{x^2}} 3^{x^2})$

$f)\ (2x+1)e^{-1/x}$ $\quad g)\ \cos(\mathrm{sen} x)\cos x$ $\quad h)\ 6\left(\dfrac{v}{v^3+1} \right)^5 \left(\dfrac{1-2v^3}{(v^3+1)^2} \right)$

$i)\ \dfrac{(x^2+1)\mathrm{sen} x - 2x\cos x}{(x^2+1)^2}$ $\quad j)\ \dfrac{x\mathrm{sen} x - (x+1)(\mathrm{sen} x + x\cos x)}{x^2 \mathrm{sen}^2 x}$

$k)\ \dfrac{3e^{3x}(1-e^{5x}) + 5e^{5x}(1+e^{3x})}{(1-e^{5x})^2}$ $\quad l)\ \dfrac{x\ln x - (x+1)(\ln x + 1)}{x^2 \ln^2 x}$

$m)\ e^{-x}(2\cos(2x) - \mathrm{sen}(2x))$ $\quad n)\ 3x^2 e^{-3x}(1-x)$ $\quad o)\ -9x^2\cos^2 x^3 \mathrm{sen} x^3$

$p)\ \dfrac{6x\ln^2(x^2+1)}{x^2+1}$ $\quad q)\ x^2(3\mathrm{tg}(4x) + 4x\sec^2(4x))$ $\quad r)\ \dfrac{\sec(x^2+2)(1+2x^2\mathrm{tg}(x^2+2))}{2\sqrt{x\sec(x^2+2)}}$

$(s)\ \dfrac{e^{2x}(1+2x)\ln(3x+1) - \dfrac{3xe^{2x}}{3x+1}}{\ln^2(3x+1)}$ $\quad (t)\ \dfrac{e^{-2x}[(1-2x)\sec(3x) - 3x\sec(3x)\mathrm{tg}(3x)]}{\sec^2(3x)}$

$(u)\ 5^{x^2}(1+2x^2\ln 5)$ $\quad (v)\ \dfrac{4x(x+1) - (x^2+1)}{2\sqrt{(x+1)^3}}$