Bifurcation and hyperbolicity for a nonlocal quasilinear parabolic problem.

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Abstract

We study a one-dimensional nonlocal quasilinear problem of the form

$$u_t = a(\|u_x\|^2)u_{xx} + \nu f(u),$$

with Dirichlet boundary conditions on the interval $[0, \pi]$, where $0 < m \leq a(s) \leq M$ for all $s \in \mathbb{R}^+$ and f satisfies suitable conditions. We give a complete characterization of the bifurcations and of the hyperbolicity of the corresponding equilibria. With respect to the bifurcations we extend the existing results to the case when $a(\cdot)$ is not necessarily monotone increasing and show that bifurcations may be pitchfork or saddle-node, subcritical or supercritical. We also give a complete characterization of hyperbolicity specifying necessary and sufficient conditions for its presence or absence.