Weakly Coupled Logistic Systems and their Asymptotic Behavior

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In this talk we present some recent results the weakly coupled elliptic system of logistic type,

$$\begin{cases} -\Delta u &= \lambda_1 u - |u|^{p-2}u + \beta |u|^{\frac{p}{2}-2}u|v|^{\frac{p}{2}-1}v \text{ in } \Omega, \\ -\Delta v &= \lambda_2 v - |v|^{p-2}v + \beta |u|^{\frac{p}{2}-1}u|v|^{\frac{p}{2}-2}v \text{ in } \Omega, \\ u, v &\in H_0^1(\Omega), \end{cases} \tag{LS}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with $N \geq 2$, $2 < p < 2^*$, and $\lambda_1(\Omega) < \lambda_1 \leq \lambda_2$. We say the system is competitive if $\beta < 0$ and cooperative if $\beta > 0$, for $\beta \in \mathbb{R}$.

We prove the existence and multiplicity of solutions to the problem (LS) in alternative variational frameworks, depending on the range of the parameter β. We do not rely on bifurcation or degree theory, which have been used in the literature for logistic-type problems. Instead, the novelty is to obtain min-max type solutions by exploiting the different geometry of the functional associated with the logistic problem. In case $N \geq 2$ and suitable values of p, we extend the existence results, for all β in the whole line, and possibly for the classical case $N = 3$ and $p = 4$. Furthermore, we analyze the asymptotic behavior of such solutions as $\beta \to 0$ or $\beta \to \pm\infty$. Work in collaboration with Haoyu Li (UFSCar) and Mayra Soares (UnB).

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