

Existence and Multiplicity of solutions for a class of Dirac equations

Abstract

In this talk, we present recent results on the existence and multiplicity of semiclassical solutions for nonlinear Dirac equations involving continuous potentials and general nonlinear self-couplings.

In the first part, we consider a class of nonlinear Dirac equations with a small parameter $\varepsilon > 0$, where the nonlinearity depends on the modulus of the solution. Under mild assumptions, we prove that the number of solutions is at least the number of global minima of the potential function V , provided ε is sufficiently small.

In the second part, assuming that the external potential attains a local minimum, we relate the topological complexity of the set of minimum points to the multiplicity of solutions. The main tools include variational methods and the Lusternik–Schnirelmann category, combined with a penalization technique to handle the strongly indefinite nature of the associated energy functional.

Both problems are motivated by the rich structure and analytical challenges posed by the Dirac operator, particularly in the semiclassical regime. Our results contribute to the understanding of nonlinear spinorial models by establishing existence and multiplicity theorems in settings with minimal regularity and without symmetry assumptions.