ABOUT AN ALT-CAFFARELLI-FRIEDMAN MONOTONICITY FORMULA IN THE HEISENBERG GROUP

The Alt-Caffarelli-Friedman monotonicity formula is a classical tool in free boundary problems, developed by W. Alt, L. Caffarelli, and A. Friedman in [1]. This property turned out to be fundamental since it allows to prove the optimal Lipschitz regularity for solutions of Bernoulli-type two-phase free boundary problems.

In this talk, we deal with an Alt-Caffarelli-Friedman monotonicity formula in the sub-Riemannian setting of Heisenberg group. First, we recall the main elements of this non-Euclidean framework. Next, we discuss the existence of such a formula in this context. In particular, we provide an explicit example in which it does not exist. Our method takes inspiration from the work [3]. We finally briefly mention some alternative ways to face this question and related properties. The talk is based on joint work with F. Ferrari, see [2].

REFERENCES

[1] H. W. Alt, L. A. Caffarelli, A. Friedman, *Variational problems with two phases and their free boundaries*, Trans. Amer. Math. Soc. 282 (1984), no. 2, 431–461.
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