GEOMETRIA GENERALIZADA

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"Trabalhar com $TM \oplus T^*M$"

$u \in \Gamma(TM \oplus T^*M)$

$u = X + \xi$ $\quad v = Y + \eta$

vetor $\quad$ 1-forma

$n = \dim M$

$\langle u, u \rangle = \xi(X)$ $\quad$ $\boxed{\langle u, v \rangle = \frac{1}{2}\left(\xi(Y) + \eta(X)\right)}$

$\langle X - \xi, X - \xi \rangle = -1$

. bilinear
. simétrico
. não-degenerado
. ~~positivo-definido~~ $\quad$ assinatura (n,n)

$N(X,Y) = J$

. Colchete de Dorfman

$[X+\xi, Y+\eta] = [X,Y] + \mathcal{L}_X \eta - i_Y d\xi$

vetor $\quad$ Colchete de Lie $\quad$ Derivada de Lie $\quad$ 1-forma

. não é antissimétrico
. Identidade de Jacobi
falha

Fibrato tangente TM

$$\mathfrak{X}(M) \times \Omega^\bullet(M) \longrightarrow \Omega^\bullet(M)$$
$$(X, \alpha) \longmapsto i_X \alpha$$

$$i_{[X,Y]} = \mathcal{L}_X \circ i_Y - i_Y \circ \mathcal{L}_X \qquad \left(\mathcal{L}_X = d \circ i_X + i_X \circ d \right)$$

$$\boxed{= d \circ i_X \circ i_Y + i_X \circ d \circ i_Y - i_Y \circ d \circ i_X - i_Y \circ i_X \circ d}$$

$TM \oplus T^*M$

$$\Gamma(TM \oplus T^*M) \times \Omega^\bullet(M) \longrightarrow \Omega^\bullet(M)$$
$$((X+\xi), \alpha) \longmapsto (X+\xi) \cdot \alpha = i_X \alpha + \xi \wedge \alpha$$

$$[X+\xi, Y+\eta] \cdot \alpha = \ldots$$

. Estruturas complexas $J : TM \to TM$ $J^2 = -Id$

$E \subset TM$ i-autofibrado de J. $[E, E] \subset E$ integrável.

$$\mathcal{J}_J : TM \oplus T^*M \to TM \oplus T^*M$$

$$\mathcal{J}_J := \left(\begin{array}{c|c} -J & 0 \\ \hline 0 & J^* \end{array} \right)$$

. $\mathcal{J}_J^2 = -Id$ "complexa"

. $\mathcal{J}_J^* = -\mathcal{J}_J$ "antissimétrica"

$E_{\mathcal{J}}$ i-autofibrado de $\mathcal{J}_J$ $[E_{\mathcal{J}}, E_{\mathcal{J}}] \subset E_{\mathcal{J}}$ integrabilidade.

. Estruturas simpléticas $\underline{\omega \in \Omega^2(M)}$ $\boxed{d\omega = 0}$

$$\begin{aligned} \omega : TM &\to T^*M \\ X &\mapsto \omega(X, \cdot) \end{aligned}$$

$$\mathcal{J}_\omega : TM \oplus T^*M \to TM \oplus T^*M$$

$$\mathcal{J}_\omega := \begin{pmatrix} 0 & -\omega^{-1} \\ \omega & 0 \end{pmatrix}$$

- $\mathcal{J}_\omega^* = -\mathcal{J}_\omega$ "antissimétrica"
- $\mathcal{J}_\omega^2 = -Id$ "complexa"

E_ω i-autofibrado de $\mathcal{J}_\omega$ $\quad d\omega = 0 \Leftrightarrow \underline{[E_\omega, E_\omega] \subset E_\omega}$

Uma estrutura complexa generalizada em M é

$\mathcal{J} : TM \oplus T^*M \to TM \oplus T^*M$

- $\mathcal{J}^* = -\mathcal{J}$
- $\mathcal{J}^2 = -\mathcal{J}$
- $[E, E] \subset E$

E i-autofibrado de $\mathcal{J}$.

Métricas Riemannianas g

$$\mathcal{G} : TM \oplus T^*M \to TM \oplus T^*M$$

$$\mathcal{G} := \begin{pmatrix} 0 & g^{-1} \\ g & 0 \end{pmatrix}$$

$\langle \mathcal{G}u, u \rangle > 0$

$\mathcal{G} : TM \oplus T^*M \to TM \oplus T^*M$ métrica Riemanniana generalizada

- ortogonalidade
- auto-adjunta
- positiva-definida

$\Rightarrow \mathcal{G}^2 = Id$

$\Rightarrow C_\pm \subset TM \oplus T^*M$ ± 1-autofibrado

$\Rightarrow \langle \cdot, \cdot \rangle$ é $\pm$ definido em $C_\pm$

Métrica generalizada $\Leftrightarrow$ $C_+ \subset TM \oplus T^*M$ subfibrado + definido maximal.

$C_\pm$ é gráfico de um mapa $TM \to T^*M$

$$\underline{\otimes^2 T^*M} = \Omega^2 T^*M \oplus \mathrm{Sym}^2 T^*M$$

$$C_\pm = \{X + B(X,\cdot) \pm g(X,\cdot) \, ; \, X \in TM\}$$

$B \in \Omega^2(M)$ $\quad g$ métrica

métrica generalizada $\Leftrightarrow$ $\underline{\text{métrica Riemanniana}}$ g + $\underline{B \text{ 2-forma}}$

$$\mathcal{G} = \begin{pmatrix} -g^{-1}B & g^{-1} \\ g - Bg^{-1}B & Bg^{-1} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ B & 1 \end{pmatrix}} \underbrace{\begin{pmatrix} 0 & g^{-1} \\ g & 0 \end{pmatrix}} \underbrace{\begin{pmatrix} 1 & 0 \\ -B & 1 \end{pmatrix}}$$

Obrigado!