Contação de histórias e um pouco de "classificações" em ambientes algébricos

Manuela da Silva Souza

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Título antigo

Álgebras de Jordan de dimensão 2: classificação e teoria de identidades polinomiais

Defesa de doutorado 2013 (Unicamp)

Época de escola (CERS - Salvador - BA)

Escola de Álgebra 2010 (UnB)

Defesa de Mestrado de Joselma (2015)

Defesa de Pedro Henrique (2022)

Paper em colaboração (2022)

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The algebra of 2 × 2 upper triangular matrices as a commutative algebra: Gradings, graded polynomial identities and Specht property ☆

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A R T I C L E   I N F O

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A B S T R A C T

Let UJ_2 be the Jordan algebra of 2×2 upper triangular matrices. This paper is devoted to continue the description given by recent works about the gradings and graded polynomial identities on $UJ_2(K)$ when K is an infinite field of characteristic 2. Due to the definition of Jordan algebras in terms of the commutative and Jordan identities being unsuitable in characteristic 2, we decided to study the gradings of the non-associative commutative algebra of 2×2 upper triangular matrices $UT_2 = (UT_2(K), \circ)$ with the product $x \circ y = xy + yx$. More precisely, fixed K a field of characteristic 2, we classify the gradings of $(UT_2(K), \circ)$ and also, given an arbitrary grading, we calculate the generators of the ideals of graded identities and give a positive answer to the Specht property for the variety of commutative algebras generated by $(UT_2(K), \circ)$ in each grading when K is infinite.

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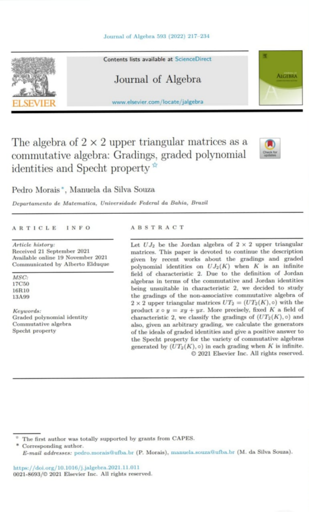

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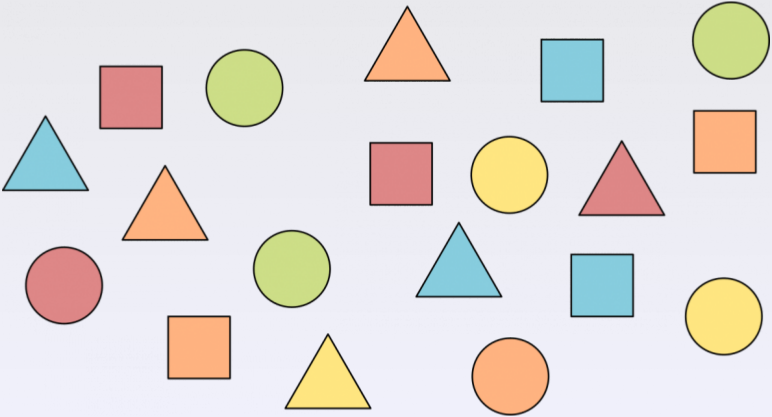
0021-8693/

Janara: a minha mais nova (doutoranda em Matemática)

PAPIC-EF: a matemática como aliada da luta antirracista

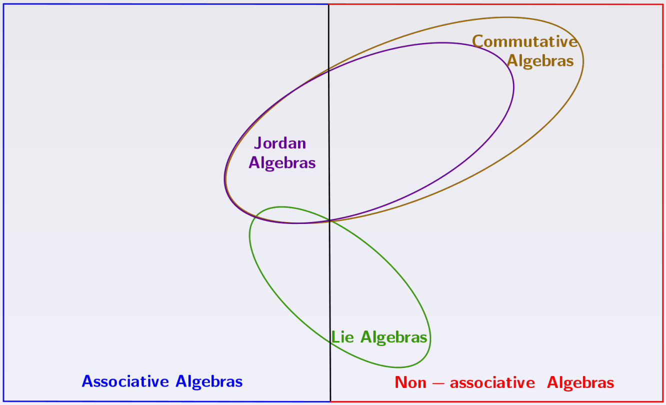
Vamos falar de classificações (no plural) de objetos



Classificações importantes nessa palestra

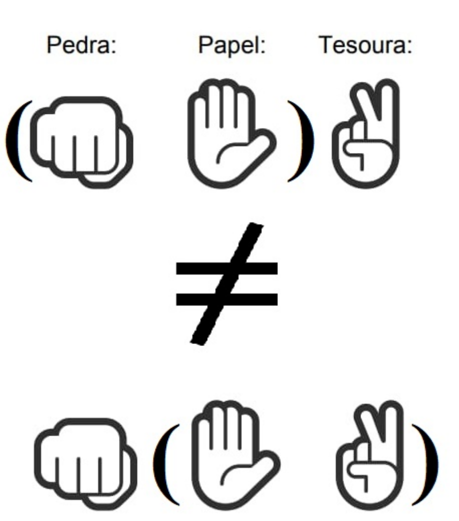
ISOMORFISMO (mais forte) e **PI-EQUIVALÊNCIA** (mais fraco)

Álgebra := espaço vetorial com um produto bilinear



Álgebra : $=$ F-álgebra (sobre um corpo finito ou infinito F).

Pedra-papel-tesoura e as operações não associativas



Teoria de identidades polinomiais (PI-teoria): principais ingredientes

INGREDIENTES:

- Um corpo F (infinito, finito, $\mathrm{char}(F) = 0$, $\mathrm{char}(F) = p > 0$).
- Uma classe de álgebras (em geral, uma variedade de álgebras).
- Noção de polinômio nessa classe (Álgebra livre na classe)
- Noção de identidade polinomial.

Álgebras de Jordan

Uma álgebra A munida com uma multiplicação denotada por $\circ$ é dita uma **álgebra de Jordan**, se para todo a, $b \in A$,

$$a \circ b = b \circ a$$

$$(a^2, b, a) = (a^2 \circ b) \circ a - a^2 \circ (b \circ a) = 0 \text{ (identidade de Jordan)}$$

$$(a_1, a_2, a_3) := (a_1 \circ a_2) \circ a_3 - a_1 \circ (a_2 \circ a_3).$$

- Álgebras de Jordan foram introduzidas por Pascual Jordan (1933) para formalizar noções da mecânica quântica.

Identidades polinomiais para álgebras de Jordan

Um polinômio de Jordan nas variáveis $X = \{x_1, x_2, \ldots\}$ $f = f(x_1, \ldots, x_k)$ é **uma identidade polinomial para a álgebra de Jordan** J se para todo a_1, ..., $a_k \in J$, $f(a_1, \ldots, a_k) = 0$.

Exemplo

Toda álgebra associativa (na classe das álgebras de Jordan) satisfaz o polinômio

$$(x_1, x_2, x_3) = (x_1x_2)x_3 - x_1(x_2x_3).$$

Dizemos que $\mathrm{Id}(J) = \{\text{identidades polinomiais para } J\}$ é o T-ideal de J. As álgebras J_1 e J_2 são **PI-equivalentes** se $\mathrm{Id}(J_1) = \mathrm{Id}(J_2)$.

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J álgebra de Jordan, $\dim(J) = 2$, F inifinito, $\mathrm{char}(F) \neq 2$

Em 2023, D. Diniz, D. Gonçalves, V. Silva, M.S. provaram

J	$\mathrm{Id}(J)$
N	$\langle x_1 x_2 x_3 \rangle^T$
M	$\langle (x_1, x_2, x_3) \rangle^T$
M_λ	$\langle (x_1, x_2, x_3) \rangle^T$
D_2	$\langle (1), (2), (3) \rangle^T$

$$T(x_1, x_2, x_3, x_4) = (x_1 x_2, x_3, x_4) - x_1(x_2, x_3, x_4) - x_2(x_1, x_3, x_4), \tag{1}$$

$$S(x_1, x_2, x_3, x_4) = (x_1, x_2 x_3, x_4) - 2x_2(x_1, x_3, x_4), \tag{2}$$

$$U(x_1, x_2, x_3, x_4, x_5) = (x_1 x_2)(x_3, x_4, x_5) - 2x_1 x_2(x_3, x_4, x_5). \tag{3}$$

M e M_λ são PI-EQUIVALENTES porém NÃO ISOMORFAS.

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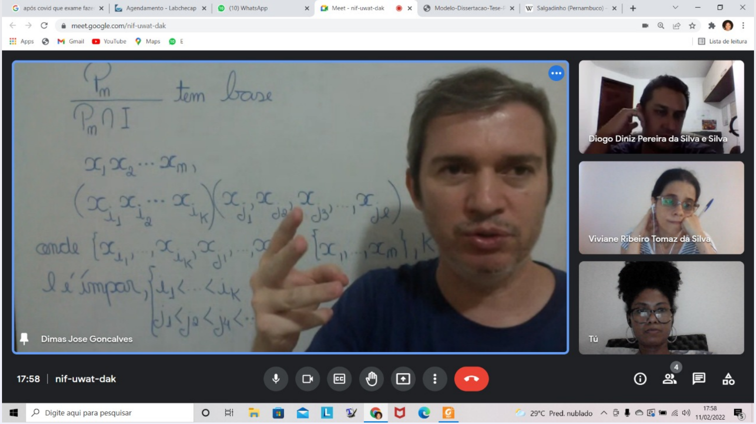
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Viviane Ribeiro Tomaz da Silva
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Dimas José Gonçalves
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J álgebra de Jordan, $\dim(J) = 2$, F finito, $\mathrm{char}(F) \neq 2$

PI-EQUIVALÊNCIA $\Leftrightarrow$ ISOMORFISMO

Referências

ELSEVIER
Linear Algebra and its Applications
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Two-dimensional Jordan algebras: Their classification and polynomial identities

Diogo Diniz [a], Dimas José Gonçalves [b], Viviane Ribeiro Tomaz da Silva [c], Manuela Souza [d]

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Abstract

Let F be an arbitrary field of characteristic different from 2. We classify the two-dimensional power-associative commutative algebras over F. As consequence, we obtain a classification of two-dimensional Jordan algebras over F and prove that there exists, up to isomorphism, a unique two-dimensional nonassociative Jordan algebra. The construction of this algebra can be generalized naturally to produce a Jordan algebra D with an arbitrary dimension. If F is infinite, we determine a finite basis for the

THE ISOMORPHISM PROBLEM IN THE CONTEXT OF PI-THEORY FOR TWO-DIMENSIONAL JORDAN ALGEBRAS

DIOGO DINIZ, DIMAS JOSÉ GONÇALVES, VIVIANE RIBEIRO TOMAZ DA SILVA, AND MANUELA DA SILVA SOUZA

ABSTRACT. Let F be a field of characteristic different from 2. Small-dimensional Jordan algebras over F have been extensively studied and such two-dimensional algebras have been classified. In the present paper we show that any two-dimensional Jordan algebras over a finite field are isomorphic if and only if they satisfy the same polynomial identities (the opposite happens in the case where F is infinite, even an algebraically closed field). In order to do this, we take a careful look at the classification of two-dimensional Jordan algebras when F is a finite field and determine a finite generating set for the T-ideal of all their polynomial identities. Linear bases for the corresponding relatively free algebras are also determined.

Keywords: Jordan algebras, polynomial identities, isomorphism, finite fields.
2020 AMS MSC Classification: 17C05, 16R10, 17A05, 17A30.

1. INTRODUCTION

Throughout the paper, F is a field of characteristic different from 2 and all algebras are over F.

Jordan algebras were introduced by the German physicist Pascual Jordan, see [18], however the term "Jordan algebras" was introduced by A. Albert in [1]. These algebras have been studied since then and have connections with other areas of Mathematics such as Differential Geometry, Functional Analysis and Projective

"Quando a mulher negra se movimenta, toda a estrutura da sociedade se movimenta com ela." (Angela Davis)

Obrigada! (manuela.dss@gmail.com)