Some boundedness results for the Prabhakar integral operator

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Abstract

The Prabhakar fractional integral represents one of the most significant generalizations in the field of fractional calculus. This operator contains the three-parameter Mittag-Leffler function $E^{\gamma}_{\alpha,\beta}(z)$ in the kernel and thus it unifies and extends several classical fractional integrals, including those of Riemann–Liouville, Saigo, and Erdélyi-Kober. Owing to the flexibility provided by its additional parameters, the Prabhakar integral has attracted considerable attention in recent years, both from theoretical and applied perspectives.

On the theoretical side, extensive studies have addressed its analytical properties, including boundedness, compactness, and continuity on various function spaces. These investigations have clarified the role of parameter restrictions such as $\alpha \in (0, 1)$, $\gamma > 0$, and $\alpha\gamma > \beta - 1 > 0$ in ensuring the well-posedness and monotonicity of the associated kernel. From the applied viewpoint, the Prabhakar integral has found growing relevance in modeling complex phenomena in viscoelasticity, anomalous diffusion, and control theory, where memory effects play a fundamental role.

Despite the remarkable progress achieved in recent years, several open questions persist regarding the boundedness and continuity of the Prabhakar operator in classical function spaces such as $C[a, b]$, $C^1[a, b]$, and $L^p(a, b)$. The present work contributes to this line of research by establishing new results on the boundedness of the Prabhakar integral operator under broad conditions on its parameters. These results enhance the theoretical understanding of this fundamental fractional operator and provide a rigorous framework for its further applications in fractional differential equations.

This is joint work with **Yuri Luchko** (Berlin University - Germany), **Gilberto A. Mendez Cruz** and **Willy Zubiaga Vera** (UNT - PERU).