Infinity Laplacian equations with gradient activation

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Abstract

We investigate diffusion processes governed by the Infinity Laplacian, where the dynamics are activated only in regions where the gradient exceeds a fixed threshold k>0. Within the framework of viscosity solutions, we show that solutions can be characterized through comparison with cones whose slopes are strictly larger than k.

This geometric viewpoint naturally selects an active region, capturing the intrinsic degeneracy of the operator. Inside this region, we establish a monotonicity formula that enables a precise classification of blow-up limits. As a consequence, we prove uniqueness of blow-ups and differentiability at active points, which classically correspond to the set $|Du| > k$.

The model is motivated by extremal regimes in congestion phenomena, particularly in optimal transport and traffic flow, where differentiability guarantees a well-defined propagation of optimal directions.

This is joint work with **Rafael Costa** (UFRGS-Brazil).