Regularity for False Mean Curvature Inequalities

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Abstract

In this talk, we will present recent results on the regularity of viscosity solutions associated with the false mean curvature operator

$$Q_\beta u = 2\langle D^2 u \nabla u, \nabla u\rangle + \beta \Delta u.$$

In general, inequalities involving fully nonlinear operators provide only limited information about the differentiability of the solutions. We will show that, for the false mean curvature operator, there is an underlying divergence structure that allows us to obtain significantly stronger regularity results than those predicted by fully nonlinear theory. As a consequence, we prove that locally Lipschitz viscosity solutions of inequalities of the type

$$|Q_\beta u| \leq f$$

belong locally to the Sobolev spaces $W^{2,p}$ for all $p > 1$, and can be interpreted as strong solutions of an effective equation. In the case where $Q_\beta u = f$ and the source term is Hölder-regular, we will also show that the solutions are classical and belong to the class $C^{2,\alpha}$. The presentation will highlight the central ideas of the proof, including a connection between viscosity and divergence formulations, Lieberman-type estimates for bilateral inequalities, and the transition to an effective linear equation. As an application, we will discuss a recent extension of a Liouville-type theorem to locally Lipschitz viscosity solutions in half-spaces.

This is joint work with **Jefferson Abrantes** (UFCG) and **Diego Moreira** (Unibo and UFC).