APRESENTA:

With Geometry, the Universe on a Sheet of Paper

18/10/2024 às 10h00

Online via *link*
https://meet.google.com/ojm-ksyb-wcn

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Abstract. Throughout history, humanity has posed countless questions about Nature. Undoubtedly, this desire to understand the world is a hallmark of human rationality, setting us apart. As Nobel Prize-winning physicist Murray Gell-Mann once said: *Understanding the Universe, how it works, where it comes from, and where it is going, is the most important challenge in the history of humanity.* We experience Nature through our senses, but these have clear limitations (for instance, we can only perceive certain wavelengths, which we call colors). Even so, many aspects of the world are perceived similarly by everyone (for example, that the shortest path between two points is a straight line, or that the length of one side of a triangle is always less than the sum of the other two sides). These common perceptions of the objects around us are abstracted and formalized into the science of Euclidean geometry. This very old discipline, more than twenty-four centuries old, is closely tied to intuition, what is drawn and the drawing itself seems to align perfectly. For a long time, it was considered the only possible geometry, until the discovery of the so-called non-Euclidean geometries, which arose in part from debates about Euclid's famous fifth postulate.

Unfortunately, Euclidean geometry is not suitable for understanding the Universe, as it is four-dimensional, incorporating time, and non-Euclidean. We tend to believe that everyone perceives space and time in the same way, but, amazingly, this is not true! Each of us has a private physical space that changes as time passes on our individual clocks. Paraphrasing Heraclitus: *no one experiences the same physical space on two different occasions.* Moreover, one person's private physical space is different from that of another. The relativity of each person's physical space can be seen as an individual perception of the spacetime universe. However, knowledge of many of these private three-dimensional spaces does not necessarily lead to an understanding of the universe as a whole. While everyone naturally uses Euclidean geometry in their relative physical space, it does not seem adequate for understanding the spacetime universe as a whole. The appropriate geometry is four-dimensional Lorentzian geometry, named after H.A. Lorentz (1853–1928).

Beyond the reasons mentioned earlier, the key lies in the concept of *curvature*. In each geometry, curvature (primarily developed by Gauss and Riemann) allows us to distinguish between

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different spaces from within. For example, using curvature, a two-dimensional being living on a sphere or an ellipsoid would be able to determine with certainty which surface it was on, without ever needing to leave it.

When there are sources of gravity or electromagnetic radiation, a certain curvature is produced in spacetime.This was Albert Einstein's brilliant insight in establishing the Theory of Relativity. Einstein postulated that mass and radiation produce the curvature of spacetime through a PDE, which we now call the Einstein field equation. Each region of the Universe can be understood as a 4-dimensional continuum, where a solution to the Einstein field equation is defined. The unknown in this PDE is a Lorentzian metric, which in relativity plays a role analogous to the gravitational potential produced by mass density in classical gravitation, as described by the Poisson equation.

Lorentzian geometry is well-founded, but it is difficult to visualize; figures are perceived in a way far removed from our Euclidean intuition. At the beginning, the role of Lorentzian geometry was explain certain physical phenomena. While this is undoubtedly important, what is truly surprising are the *predictions* that geometry makes about the behavior of the Universe as a whole, both in the future and the distant past. Relativistic cosmology is an active area of research pursued by many scientists and holds significance at various levels of human knowledge.

In 1922, A. Friedmann and, independently in 1927, G. Lemaître, discovered *on paper*, using Einstein's equation, that the Universe should be expanding. Einstein was astonished by the mathematical argument that led to this physical interpretation, despite the restrictive nature of the universe model used. However, he was initially hesitant to accept this possibility, perhaps echoing his famous remark: *The more mathematics resembles reality, the less certain it is; the more certain it is, the less it resembles reality.*

Then, in 1929, the renowned astronomer E. Hubble experimentally confirmed the expansion of galaxies through observations made with his telescope. On today, Lemaître has been considered as co-autor of the Hubble experimental law and this is now called the Hubble-Lemaître law.

Lemaître, through reasoning based on *continuity*, also theorized: *if the Universe is expanding today, then in the past it must have been much smaller, much denser, and condensed into a "primitive atom."* The process by which the Universe emerged from this primitive atom is known as the Big Bang (some believed this was a scientific explanation for the beginning of the Universe). However, Lemaître's argument required experimental confirmation, which came in 1965 when A. Penzias and R. Wilson discovered the cosmic microwave background radiation, as predicted by Gamow. In 1978, they were awarded the Nobel Prize in Physics for this discovery.

Starting in the 1970s, a scientific revolution took place with the application of highly sophisticated techniques from Lorentz Geometry in Cosmology. S. Hawking and R. Penrose established, in a general framework (unlike Lemaître, who used a specific geometric model for the Universe), the existence of the Big Bang. In essence, they proved geometric theorems with the physical interpretation that, under reasonable physical conditions, a spacetime universe must have a singularity in its past, which is understood as the initial moment of the Universe. However, the real novelty of their research, was proving, using Lorentzian geometry, that stars with sufficient mass collapse under their own gravity, forming black holes, extending an important but not too much known result of A.K. Raychaudhuri in 1955). These are regions of the Universe where gravity is so strong that not even light can escape, making them invisible to us, hence the name black holes. In this talk we will also explain a result of Hawking on existence of the Big Bang and another one of Raychaudhuri on existence of black holes.

In a very brief summary of the hundred-year relationship between the Theory of Relativity and Lorentzian geometry, it should be noted that Lorentzian geometry now stands on its own as a field of great interest from a mathematical perspective. Nevertheless, it remains highly gratifying to consider the potential physical applications of certain selected research topics within this area.